Exercises

Boundary Conditions

in lattice Boltzmann method

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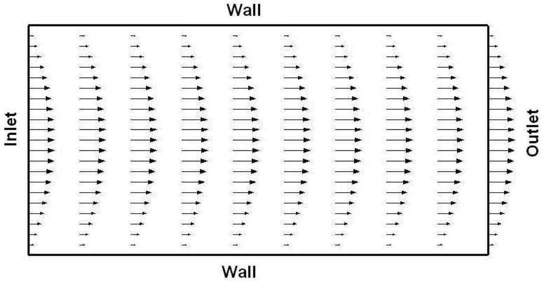
Exercise I:

Poiseuille flow with bounceback walls



Setting the problem

- Governing equation:

$$\nu \frac{\partial u_x}{\partial y} = -a_x$$

- Reynolds number:

$$Re = \frac{U_{max}(y_{top} - y_{bottom})}{\nu}$$

- Velocity solution:

$$u_x(y) = -\frac{1}{2\nu} a_x (y - y_{bottom})(y - y_{top})$$

Setting the problem

- BounceBack walls location:

$$\begin{cases} y_{bottom} = 0.5 \\ y_{top} = NY - 0.5 \end{cases}$$

- Body force (acceleration) magnitude:

$$a_x = \frac{8 u_{max} \nu}{(y_{top} - y_{bottom})^2}$$

Input parameters

- $\mathbf{N_{steps}}$ → Number of time steps to achieve steady-state
- **NY** → Number of nodes along the channel height
- **Re** → Flow Reynolds number
- ω → Relaxation frequency (LBM parameter)
- $\mathbf{U_{max}}$ → Maximum velociy (proportional to Mach number)

Questions

- **Question 1:** Implement Bounceback BC
- **Question 2:** Fix $Re = 10$ and $\omega = 1$
  - **Question 2.1** For $NY = 16$, what is U_{max} and $\|L_2\|_\infty$?
  - **Question 2.2** For $NY = 32$, what is U_{max} and $\|L_2\|_\infty$?
  - **Question 2.3** Estimate the convergence rate

  Hint: $\alpha = \ln\left(\frac{\|L_2\|_\infty(u_M)}{\|L_2\|_\infty(u_N)}\right)/\ln(\frac{N}{M})$

Questions

- **Question 3** Fix $Re = 10$ and $NY = 16$
  - **Question 3.1** What is $\|L_2\|_\infty$ for $\omega = 1$, $\omega = 1.7$, $\omega = 0.5$, $\omega = 0.2$ and $\omega = \frac{32}{20+\sqrt{208}}$?
  - **Question 3.2** For each of the above ω values, compute the momentum imbalance $\Delta \mathbf{F}$ between the applied force and the friction force at walls. Comment the results.

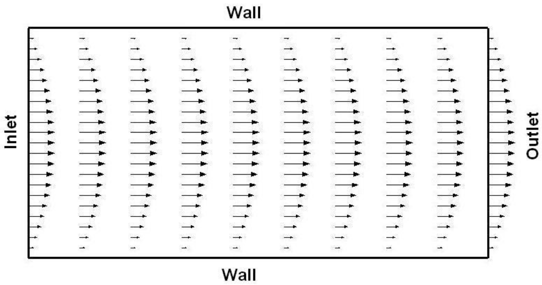
Hint: $\mathbf{F}_{wall} + \mathbf{F}_{prop} = \Delta \mathbf{F}$

$$\mathbf{F}_{wall} = -2\frac{\Delta x}{\Delta t} \sum_{\mathbf{x}_b \in S} \sum_{\alpha} \mathbf{c}_\alpha \tilde{f}_\alpha(\mathbf{x}, t)$$

$$\mathbf{F}_{prop} = \sum_{\mathbf{x}_b \in V} a_x$$

Exercise II:

Poiseuille flow with Zou He walls



Setting the problem

- Governing equation:

$$\nu \frac{\partial u_x}{\partial y} = -a_x$$

- Reynolds number:

$$Re = \frac{U_{max}(y_{top} - y_{bottom})}{\nu}$$

- Velocity solution:

$$u_x(y) = -\frac{1}{2\nu} a_x (y - y_{bottom})(y - y_{top})$$

Setting the problem

- Zou He walls location:

$$\begin{cases} y_{bottom} = 1 \\ y_{top} = NY \end{cases}$$

- Body force (acceleration) magnitude:

$$a_x = \frac{8 u_{max} \nu}{(y_{top} - y_{bottom})^2}$$

Input parameters

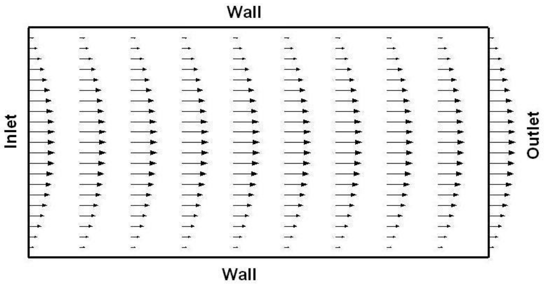
- $\mathbf{N_{steps}} \rightarrow$ Number of time steps to achieve steady-state
- $\mathbf{NY} \rightarrow$ Number of nodes along the channel height
- $\mathbf{Re} \rightarrow$ Flow Reynolds number
- $\omega \rightarrow$ Relaxation frequency (LBM parameter)
- $\mathbf{U_{max}} \rightarrow$ Maximum velociy (proportional to Mach number)

Questions

- **Question 1:** Implement Zou He BC
- **Question 2:** Fix $Re = 10$ and $\omega = 1$
  - **Question 2.1** For $NY = 16$, what is U_{max} and $\|L_2\|_\infty$?
  - **Question 2.2** For $NY = 32$, what is U_{max} and $\|L_2\|_\infty$?
  - **Question 2.3** Discuss the solutions accuracy
- **Question 3** Fix $Re = 10$ and $NY = 16$
  - **Question 3.1** What is $\|L_2\|_\infty$ for $\omega = 1$, $\omega = 1.7$, $\omega = 0.5$, $\omega = 0.2$?

Exercise III:

Developing Poiseuille flow with Zou He walls and inlet/outlet BC



Setting the problem

- Velocity inlet solution:

$$(u_x)_{in}(y) = \frac{1}{12\nu} a_x (y_{bottom}^2 - 2 y_{bottom} y_{top} + y_{top}^2)$$

where the driving force term is:

$$a_x = \frac{8 u_{max} \nu}{(y_{top} - y_{bottom})^2}$$

- Velocity outlet solution:

$$\frac{\partial (u_x)_{out}}{\partial y} = 0$$

Input parameters

- $\mathbf{N_{steps}} \rightarrow$ Number of time steps to achieve steady-state
- $\mathbf{NY} \rightarrow$ Number of nodes along the channel height
- $\mathbf{Re} \rightarrow$ Flow Reynolds number
- $\omega \rightarrow$ Relaxation frequency (LBM parameter)
- $\mathbf{U_{max}} \rightarrow$ Maximum velociy (proportional to Mach number)

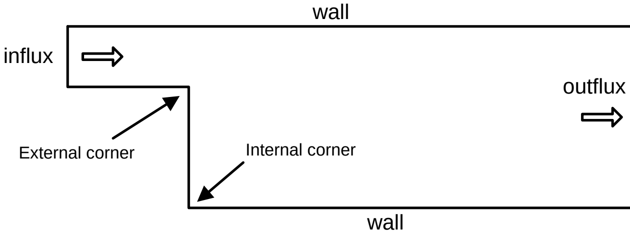
Questions

- **Question 1:** Implement Zou He BC at inlet and outlet
- **Question 2:** Fix $Re = 10$ and $\omega = 0.9$
  - **Question 2.1** For $NX = 22$ and $NY = 22$, what is $||L_2||_\infty$?
  - **Question 2.2** For $NX = 32$ and $NY = 32$, what is $||L_2||_\infty$?
  - **Question 2.3** For $NX = 52$ and $NY = 52$, what is $||L_2||_\infty$?
- **Question 3** Estimate graphically the entrance length L_h for the fully developed condition and compare with semi-analytical solution $\frac{L_h}{W} = 0.05 \times Re$

  Hint: In command window of MATLAB execute:
  plot(1:NX,ux(:,round(NY/2)))

Exercise IV:

Backward facing step flow using Zou He wall boundaries



Setting the problem

- **Remark I**: A comment on the external corner

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  - → Zou He framework does not apply for external corners, the problem remains overspecified, *e.g.* only unknowns are ρ and f_6
  - → **Solution:** use bounceback instead

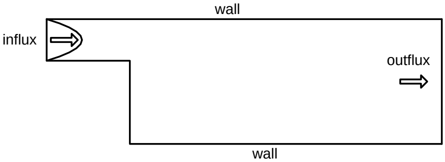
Setting the problem

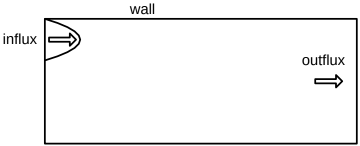
- **Remark I**: A comment on the external corner
  - → Zou He framework does not apply for external corners, the problem remains overspecified, *e.g.* only unknowns are ρ and f_6
  - → **Solution:** use bounceback instead
  - → **Differences?** On corners **Zou He reduces to bounceback** if $\mathbf{u} = \mathbf{0}$

Setting the problem

- **Remark II**: A comment on problem geometry

Geometry may be simplified if a parabolic profile is used at inlet





Setting the problem

- Influx boundary condition:

$$u_x = -\frac{1}{2\nu} a_x (y - y_{bottom})(y - y_{top\,inlet})$$

$$u_y = 0$$

where:

→ $y_{top\,inlet} - y_{bottom} =$ width of the influx boundary

→ $a_x = \frac{8 u_{max} \nu}{(y_{top\,inlet} - y_{bottom})^2}$

- Outflux boundary condition:

$$\frac{\partial u_x}{\partial x} = 0$$

$$u_y = 0$$

Input parameters

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- **Re** → Flow Reynolds number
- ω → Relaxation frequency (LBM parameter)
- $\mathbf{U_{max}}$ → Maximum velociy (proportional to Mach number)

Input parameters

- Influx width:

$$y_{top\ inlet} = \frac{Re\nu}{U_{max}} + y_{bottom}$$

- Geometry width

$$Influx\ ratio = \frac{y_{top\ inlet} - y_{bottom}}{y_{top} - y_{bottom}}$$

- Geometry length

$$L_x = L_y = y_{top} - y_{bottom}$$

Questions

- **Question 1:** Implement Zou He BC at left boundary, *i.e.* {influx $\bigcup$ left wall}
- **Question 2:** Implement Zou He BC at right boundary, *i.e.* outflux
- **Question 3:** Implement Zou He BC at bottom left and top right corners
- **Question 4:** Set $Re = 20$, $U_{max} = 0.1$ and $\omega = 0.8$
  - **Question 4.1** Set three *Influx ratio* values, *e.g.* $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{1}{1.2}$?
  - **Question 4.2** Discuss the size of recirculation patterns and the location of the outflow boundary?

Exercise V:

Flow around circular cylinder