Boundary Conditions
in lattice Boltzmann method

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Outline

1 Introduction
   - Boundary Value Problems

2 Motivation
   - Navier-Stokes Boundary Conditions

3 Lattice Boltzmann Boundary Conditions
   - Problem definition
   - Boundaries in LBM
   - Particulate dynamics
   - Using Chapman-Enskog

4 Summary
Outline

1. Introduction
   - Boundary Value Problems

2. Motivation
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3. Lattice Boltzmann Boundary Conditions
   - Problem definition
   - Boundaries in LBM
   - Particulate dynamics
   - Using Chapman-Enskog

4. Summary
Definitions

Boundary Value Problem \{ \text{Partial Differential Equation} \}
\quad \text{Boundary Condition}
Definitions

- Boundary Value Problem

\[ \begin{align*}
\text{Partial Differential Equation} \\
\text{Boundary Condition}
\end{align*} \]

e.g. Poisson equation:

\[ \begin{align*}
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} &= f(x, y), \quad \text{in } \Omega \\
\varphi &= \varphi_b, \quad \text{on } \partial \Omega
\end{align*} \]
Definitions

- Types of Boundary Conditions
Definitions

- Types of Boundary Conditions

  - Dirichlet Boundary Condition
    \[ \varphi = \varphi_b \text{ on } \partial \Omega \]
Types of Boundary Conditions

→ Dirichlet Boundary Condition

\[ \varphi = \varphi_b \quad \text{on } \partial \Omega \]

→ Neumann Boundary Condition

\[ \hat{n} \cdot \nabla \varphi = \frac{\partial \varphi}{\partial n} = \varphi_b \quad \text{on } \partial \Omega \]
Definitions

- Types of Boundary Conditions

  → Dirichlet Boundary Condition

  \[ \varphi = \varphi_b \quad \text{on } \partial \Omega \]

  → Neumann Boundary Condition

  \[ \hat{n} \cdot \nabla \varphi = \frac{\partial \varphi}{\partial n} = \varphi_b \quad \text{on } \partial \Omega \]

  → Robin Boundary Condition

  \[ g \varphi + h \frac{\partial \varphi}{\partial n} = \varphi_b \quad \text{on } \partial \Omega \]
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4 Summary
Definitions

- **Steady isothermal and incompressible Navier-Stokes equations**

\[
\left\{
\begin{align*}
(u \cdot \nabla)u &= -\nabla p + \nu \Delta u + a \\
\nabla \cdot u &= 0
\end{align*}
\right. \quad \text{in } \Omega
\]
Definitions

- Steady isothermal and incompressible Navier-Stokes equations

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\begin{align*}
(u \cdot \nabla)u &= -\nabla p + \nu \Delta u + a \\
\nabla \cdot u &= 0
\end{align*}
\text{ in } \Omega
\]

- Boundary Condition on solid walls

\[ u = u_b \quad \text{on } \partial \Omega \]
Definitions

- Steady isothermal and incompressible Navier-Stokes equations
  \[
  \begin{aligned}
  (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \nu \Delta \mathbf{u} + \mathbf{a} \\
  \nabla \cdot \mathbf{u} &= 0
  \end{aligned}
  \] in \( \Omega \)

- Boundary Condition on solid walls
  \[ \mathbf{u} = \mathbf{u}_b \text{ on } \partial \Omega \]

- Boundary Condition on fluid boundaries
  \[ \mathbf{u} = \mathbf{u}_{in} \text{ on } \partial \Omega \]

or
\[
\begin{aligned}
  -p + \nu \frac{\partial u_n}{\partial n} &= (F_n)_{in} \text{ on } \partial \Omega \\
  \nu \frac{\partial u_t}{\partial n} &= (F_t)_{in}
  \end{aligned}
\]
Definitions

- Periodic or Cyclic Boundary Conditions

\[ u(x, y) = u(x + L, y) \quad \text{in } \Omega \quad \Rightarrow \quad u_{in} = u_{out} \quad \text{on } \partial \Omega \]
Definitions

- Periodic or Cyclic Boundary Conditions

\[ u(x, y) = u(x + L, y) \quad \text{in } \Omega \quad \Rightarrow \quad u_{in} = u_{out} \quad \text{on } \partial \Omega \]

- Symmetry Boundary Conditions

\[ u \cdot n = 0 \quad \text{and} \quad \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial \Omega \]
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4 Summary
Introduction and motivation

- Lattice Boltzmann method (LBM)

\[ f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \quad \text{in } \Omega \]
Introduction and motivation

Lattice Boltzmann method (LBM)

\[ f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \quad \text{in } \Omega \]

Hydrodynamic Boundary Conditions in LBM
Introduction and motivation

- Lattice Boltzmann method (LBM)

\[ f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \quad \text{in } \Omega \]

- Hydrodynamic Boundary Conditions in LBM

→ Solution on \( \partial\Omega \) is specified for \( f_\alpha \) and NOT for \( \{\rho, u, \Pi\} \)
Introduction and motivation

- **Lattice Boltzmann method (LBM)**

  \[
  f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \quad \text{in } \Omega
  \]

- **Hydrodynamic Boundary Conditions in LBM**

  → Solution on $\partial \Omega$ is specified for $f_\alpha$ and **NOT** for $\{\rho, u, \Pi\}$

  → $f_\alpha$ set in a higher DoF system than $\{\rho, u, \Pi\}$, hence:

    - Trivial: $f_\alpha \rightarrow \{\rho, u, \Pi\}$
    - Complex: $\{\rho, u, \Pi\} \rightarrow f_\alpha$
Introduction and motivation

- Lattice Boltzmann method (LBM)

  \[ f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \text{ in } \Omega \]

- Hydrodynamic Boundary Conditions in LBM

  - Solution on \( \partial \Omega \) is specified for \( f_\alpha \) and NOT for \( \{\rho, u, \Pi\} \)
  - \( f_\alpha \) set in a higher DoF system than \( \{\rho, u, \Pi\} \), hence:
    - Trivial: \( f_\alpha \rightarrow \{\rho, u, \Pi\} \)
    - Complex: \( \{\rho, u, \Pi\} \rightarrow f_\alpha \)
  - Incorrect upscaling \( \rightarrow \) Unwanted behavior, e.g. Knudsen layers
Lattice structure

- D2Q9 model

\[
\begin{align*}
\mathbf{c}_1 &= (0, 0) \\
\mathbf{c}_2 &= (1, 0) \\
\mathbf{c}_3 &= (0, 1) \\
\mathbf{c}_4 &= (-1, 0) \\
\mathbf{c}_5 &= (0, -1) \\
\mathbf{c}_6 &= (1, 1) \\
\mathbf{c}_7 &= (-1, 1) \\
\mathbf{c}_8 &= (-1, -1) \\
\mathbf{c}_9 &= (-1, -1)
\end{align*}
\]
→ **x** is a **fluid node** if \( \forall c \) so that \( x + c \Delta t \in \{\Omega \cup \partial \Omega\} \)

→ **x** is a **boundary node** if \( \exists c \) so that \( x + c \Delta t \notin \{\Omega \cup \partial \Omega\} \)
Periodic Boundary Conditions

Periodicity → $u_{in} = u_{out}$ on $\partial\Omega$
Symmetry Boundary Conditions

\[ \text{Symmetry } \rightarrow \quad u \cdot n = 0 \quad \text{and} \quad \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial \Omega \]

(also called free-slip boundary)
Bounceback Boundary Conditions

A very intuitive idea:

A hard wall reflects particles back to where they originally came from
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A hard wall reflects particles back to where they originally came from

As a result:

→ There is no flux crossing the wall, i.e. the wall is impermeable
→ There is no relative transverse motion between fluid and wall, i.e. no-slip at the wall
REMEMBER: LBM algorithm can be operated in 2 steps:

→ Collision step:

\[ \tilde{f}_\alpha(x, t) = f_\alpha(x, t) - \omega(f_\alpha - f_\alpha^{(eq)})|_{(x,t)} \]

→ Streaming step:

\[ f_\alpha(x + c_\alpha \Delta t, t + \Delta t) = \tilde{f}_\alpha(x, t) \]
The Bounceback method can be implemented following 2 reasonings:

- **Full-way bounceback:**
  - inversion of particle velocity takes place during the collision step

- **Half-way bounceback:**
  - inversion of particle velocity takes place during the streaming step
Bounceback Boundary Conditions

**Full-way Bounceback**: \( \tilde{f}_\alpha(x_b, t) = f_\alpha(x_b, t) \)
Bounceback Boundary Conditions

**Full-way Bounceback:** \( \tilde{f}_\alpha (x_b, t) = f_\alpha (x_b, t) \)

**Half-way Bounceback:** \( f_\alpha (x_f, t + \Delta t) = \tilde{f}_\alpha (x_f, t) \)
Half-way bounceback in 2D:
Bounceback Boundary Conditions: summary

**Pros**
- Mass is exactly conserved
- Stable for \(\omega\) close to 2 (i.e. for high Re)
- Local
- Flexibility in handling wall, edges, corners both in 2D and 3D
- Very simple to implement from a programming viewpoint

**Cons**
- Velocity accuracy may decrease from 2\(^{nd}\) to 1\(^{st}\)
- Pressure accuracy may decrease from 1\(^{st}\) to 0\(^{th}\)
- In SRT model momentum is not exactly conserved (viscosity dependent slip velocity), which is equivalent to say the boundary location is not exactly defined (viscosity dependent slip length)
Question:

Use the half-way bounceback scheme to find the unknown populations at $t + \Delta t$
**Question:**

Use the half-way bounceback scheme to find the unknown populations at $t + \Delta t$.

**Solution:**

![Diagram showing bounceback boundary conditions for bottom and top walls](image-url)
Momentum exchange

- Force (per unit volume):

\[ F\big|_{(t + \frac{\Delta t}{2})} = \frac{\Delta p}{\Delta t}\big|_{(t + \frac{\Delta t}{2})} = \frac{1}{\Delta t}(p(t + \Delta t) - p(t)) \]
Momentum exchange

- Force (per unit volume):

\[ F|_{(t + \frac{\Delta t}{2})} = \frac{\Delta p}{\Delta t}|_{(t + \frac{\Delta t}{2})} = \frac{1}{\Delta t}(p(t + \Delta t) - p(t)) \]
Momentum exchange

- Force (per unit volume):

\[
F \bigg|_{(t+\frac{\Delta t}{2})} = \frac{\Delta p}{\Delta t} \bigg|_{(t+\frac{\Delta t}{2})} = \frac{1}{\Delta t} (p(t + \Delta t) - p(t))
\]

- Momentum exchange (per unit volume) between the fluid/wall surface:

\[
\Delta p(x, t + \frac{\Delta t}{2}) = \sum_{\alpha} \left[ (c_{\alpha}) f_{\tilde{\alpha}}(x, t + \Delta t) - (c_{\alpha}) \tilde{f}_{\alpha}(x, t) \right]
\]
Momentum exchange

- Momentum exchange between the fluid/wall surface:

\[ \Delta p(x, t + \frac{\Delta t}{2}) = -\sum_{\alpha} \mathbf{c}_{\alpha} \left[ f_{\alpha}(x, t + \Delta t) + \tilde{f}_{\alpha}(x, t) \right] \]
Momentum exchange

- Momentum exchange between the fluid/wall surface:

\[ \Delta p(x, t + \frac{\Delta t}{2}) = - \sum_{\alpha} c_{\alpha} \left[ f_{\alpha}(x, t + \Delta t) + \tilde{f}_{\alpha}(x, t) \right] \]

**Remember:** Half-way Bounceback: \( f_{\alpha}(x_f, t + \Delta t) = \tilde{f}_{\alpha}(x_f, t) \)
Momentum exchange

• Momentum exchange between the fluid/wall surface:

\[ \Delta p(x, t + \frac{\Delta t}{2}) = - \sum_{\alpha} c_{\alpha} \left[ f_{\alpha}(x, t + \Delta t) + \tilde{f}_{\alpha}(x, t) \right] \]

Remember: Half-way Bounceback: \( f_{\alpha}(x_f, t + \Delta t) = \tilde{f}_{\alpha}(x_f, t) \)

\[ \Rightarrow \Delta p(x, t + \frac{\Delta t}{2}) = -2 \sum_{\alpha} c_{\alpha} \tilde{f}_{\alpha}(x, t) \]
Momentum exchange

- Momentum exchange between the fluid/wall surface:

\[ \Delta p(x, t + \frac{\Delta t}{2}) = - \sum_\alpha c_\alpha \left[ f_\alpha(x, t + \Delta t) + \tilde{f}_\alpha(x, t) \right] \]

Remember: Half-way Bounceback: \( f_\alpha(x_f, t + \Delta t) = \tilde{f}_\alpha(x_f, t) \)

\[ \Rightarrow \Delta p(x, t + \frac{\Delta t}{2}) = -2 \sum_\alpha c_\alpha \tilde{f}_\alpha(x, t) \]

Force on the fluid due to the wall:

\[ F(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_\alpha c_\alpha \tilde{f}_\alpha(x, t) \]
Transverse force on the fluid due to the bottom wall

\[ F_x(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_\alpha)_x \tilde{f}_\alpha(x, t) \]
Transverse force on the fluid due to the bottom wall

\[ F_x(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_\alpha) f_\alpha(x, t) \]
Transverse force on the fluid due to the bottom wall

\[ F_x(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_{\alpha})_x \tilde{f}_\alpha(x, t) \]

Transverse force by bottom wall:

\[ F_x(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} (\tilde{f}_9 - \tilde{f}_8)|_{(x,t)} \]
Normal force on the fluid due to the bottom wall

\[ F_y(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_{\alpha})_y \tilde{f}_\alpha(x, t) \]
Normal force on the fluid due to the bottom wall

\[ F_y(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_\alpha) y \tilde{f}_\alpha(x, t) \]
Normal force on the fluid due to the bottom wall

\[ F_y(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} \sum_{\alpha} (c_{\alpha})_y \tilde{f}_{\alpha}(x, t) \]
Question:
Write the formulas of the transverse and normal forces at the top wall

Remember:
Momentum exchange: Exercise

Question:
Write the formulas of the transverse and normal forces at the top wall

Solution:

- Transverse force by **top wall**:

\[ F_x(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} (\tilde{f}_6 - \tilde{f}_7)|(x,t) \]

- Normal force by **bottom wall**:

\[ F_y(x, t + \frac{\Delta t}{2}) = -2 \frac{\Delta x}{\Delta t} \sum_{x_b \in S} (\tilde{f}_3 + \tilde{f}_6 + \tilde{f}_7)|(x,t) \]
Exercise I:

Poiseuille flow with bounceback walls
Motivation

- The solution of the Navier-Stokes not only requires the no-slip velocity condition on walls but also demands these equations to be valid near the wall.
Motivation

- The solution of the Navier-Stokes not only requires the no-slip velocity condition on walls but also demands these equations to be valid near the wall.

- Taking advantage of the Chapman-Enskog expansion...

\[ f = f^{(0)}(\rho, u) + \epsilon f^{(1)}(\nabla u) + O(\epsilon^2) \]
Motivation

The solution of the Navier-Stokes not only requires the no-slip velocity condition on walls but also demands these equations to be valid near the wall.

Taking advantage of the Chapman-Enskog expansion...

\[ f = f^{(0)}(\rho, u) + \epsilon f^{(1)}(\nabla u) + O(\epsilon^2) \]

...it can be shown:

\[ f^{(0)}_\alpha = w_\alpha \left( \rho + \frac{c_\alpha}{c_s^2} \cdot u + \frac{(c_\alpha c_\alpha - c_s^2 I)}{2c_s^4} : uu \right) \]

\[ f^{(1)}_\alpha = -w_\alpha \frac{(c_\alpha c_\alpha - c_s^2 I)}{\omega c_s^2} : (\nabla u + (\nabla u)^T) \]
-> Boundary node and solid node coincide

-> Only unknown incoming populations are modified

-> Set $\rho$ or $\mathbf{u}$ in $f^{(0)}_{\alpha}(\rho, \mathbf{u})$

-> Construct $f^{(1)}_{\alpha}$ from the symmetry requirement
Zou He velocity boundary condition

Known:

\[ \rightarrow \mathbf{u} = 0 \]

\[ \rightarrow f_\alpha = (f_1, f_2, f_4, f_5, f_8, f_9) \]
Zou He velocity boundary condition

Known:
- \( u = 0 \)
- \( f_\alpha = (f_1, f_2, f_4, f_5, f_8, f_9) \)

Unknown (4 variables):
- \( \rho \)
- \( f_\alpha = (f_3, f_6, f_7) \)
Zou He velocity boundary condition

Known:

\[ \rightarrow u = 0 \]
\[ \rightarrow f_\alpha = (f_1, f_2, f_4, f_5, f_8, f_9) \]

Unknown (4 variables):

\[ \rightarrow \rho \]
\[ \rightarrow f_\alpha = (f_3, f_6, f_7) \]

3 Equations (2 linearly independent):

\[ \rightarrow \sum f_\alpha = \rho \]
\[ \rightarrow \sum c_\alpha f_\alpha = u \]
Zou He velocity boundary condition

Symmetry of $f^{(1)}_{\alpha}$ (3 equations):

→ Bounceback of non-equilibrium populations
Zou He velocity boundary condition

- Symmetry of $f^{(1)}_{\alpha}$ (3 equations):
  - Bounceback of non-equilibrium populations

- Introduce extra variable (problem overspecified):
  - Transverse momentum correction $N_{xy}$
Zou He velocity boundary condition

- Symmetry of \( f_{\alpha}^{(1)} \) (3 equations):
  - \( \rightarrow \) Bounceback of non-equilibrium populations

- Introduce extra variable (problem overspecified):
  - \( \rightarrow \) Transverse momentum correction \( N_{xy} \)

- Problem is well specified:
  - \( \rightarrow \) 6 eqs. and 6 unknowns
1) Computing $\rho$...

Population velocity set at boundary node:

$C_+ = \{c_3, c_6, c_7\}$

$C_0 = \{c_1, c_2, c_4\}$

$C_- = \{c_5, c_8, c_9\}$

Use the two velocity moments:

$\sum f_\alpha = \rho$

$\sum (c_\alpha)_y f_\alpha = u_y$
Zou He velocity boundary condition

1) Computing $\rho$...

\[\partial\Omega \quad \Omega\]

\[c_7 \quad c_3 \quad c_6\]

\[c_4 \quad c_5 \quad c_2\]

\[c_9 \quad c_9\]

\[y \quad x\]

Relate the two velocity moments:

\[
\begin{cases}
\rho = \rho_+ + \rho_0 + \rho_- \\
uy = \rho_+ - \rho_-
\end{cases}
\]
Zou He velocity boundary condition

1) Computing $\rho$...

\[ \partial \Omega \]

\[ \Gamma \]

Relate the two velocity moments:

\[ \begin{align*}
\rho &= \rho_+ + \rho_0 + \rho_- \\
uy &= \rho_+ - \rho_-
\end{align*} \]

Solution:

\[ \rho = u_y + \rho_0 + 2\rho_- \]

i.e. \[ \rho = u_y + (f_1 + f_2 + f_4) + 2(f_5 + f_8 + f_9) \]
2) Computing \( \{f_3, f_6, f_7\} \)...

- Non-equilibrium bounceback with transverse momentum correction:

\[
\begin{align*}
    f_3 - f_3^{(0)} &= f_5 - f_5^{(0)} \\
    f_6 - f_6^{(0)} &= f_8 - f_8^{(0)} + N_{xy} \\
    f_7 - f_7^{(0)} &= f_9 - f_9^{(0)} - N_{xy}
\end{align*}
\]
2) Computing \( \{f_3, f_6, f_7\} \)... 

Solution for the unknown incoming populations:

\[
\begin{align*}
    f_3 &= f_5 + \frac{2}{3}u_y \\
    f_6 &= f_8 + \frac{1}{2}(f_4 - f_2) + \frac{1}{6}u_y + \frac{1}{2}u_x \\
    f_7 &= f_9 - \frac{1}{2}(f_4 - f_2) + \frac{1}{6}u_y - \frac{1}{2}u_x
\end{align*}
\]
Zou He boundary condition

**Pros**

→ Local
→ Velocity is 2\textsuperscript{nd} order accurate
→ Pressure accuracy is (at worst) 1\textsuperscript{st} order accurate
→ In SRT model momentum is conserved (up to 2\textsuperscript{nd} order)

**Cons**

→ Unstable when $\omega \to 0$
→ Mass is not exactly conserved (2\textsuperscript{nd} order accurate)
→ Flexibility in handling wall, edges and corners or 2D and 3D domains are being modeled
→ Not so simple to implement (compared to bounceback)
Question:
Use the Zou He procedure to find $\rho$ and the unknown populations at the top wall
Zou He boundary condition: Exercise

**Question:**
Use the Zou He procedure to find $\rho$ and the unknown populations at the top wall

**Solution:**

$$\rho = -u_y + (f_1 + f_2 + f_4) + 2(f_3 + f_6 + f_7)$$

$$f_5 = f_3 - \frac{2}{3} u_y$$

$$f_8 = f_6 + \frac{1}{2} (f_2 - f_4) - \frac{1}{6} u_y - \frac{1}{2} u_x$$

$$f_9 = f_7 - \frac{1}{2} (f_2 - f_4) - \frac{1}{6} u_y + \frac{1}{2} u_x$$
Exercise II

Poiseuille flow Zou He walls
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## Bounceback vs. Zou He

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<th>Bounceback</th>
<th>Zou He</th>
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<tbody>
<tr>
<td>Boundary location</td>
<td>Halfway</td>
<td>On node</td>
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<tr>
<td>Stable</td>
<td>Yes</td>
<td>Not as stable</td>
</tr>
<tr>
<td>Velocity accuracy</td>
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</tr>
<tr>
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<tr>
<td>Mass conservative</td>
<td>Yes</td>
<td>Only 2nd</td>
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<tr>
<td>Viscosity independent</td>
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<td>Flexibility</td>
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- **Stable**: Yes, Not as stable
- **Velocity accuracy**: 2nd to 1st
- **Pressure accuracy**: 1st to 0th
- **Mass conservative**: Yes, Only 2nd
- **Viscosity independent**: Not in SRT, Yes
- **Flexibility**: Yes, Not as flexible
- **Coding simplicity**: Yes, Not as simple

LBM Workshop 2011, August 23
## Bounceback vs. Zou He

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- **Velocity accuracy**: 2nd to 1st
- **Pressure accuracy**: 1st to 0th
- **Mass conservative**: Yes
- **Viscosity independent**: Not in SRT, Yes
- **Flexibility**: Yes, Not as flexible
- **Coding simplicity**: Yes, Not as simple
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- **Bounceback**
  - Halfway
  - Yes
  - $2^{nd}$ to $1^{st}$

- **Zou He**
  - On node
  - Not as stable
  - $2^{nd}$
### Bounceback vs. Zou He

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Viscosity independent: Yes (in SRT), Not in SRT

Flexibility: Yes, Not as flexible

Coding simplicity: Yes, Not as simple
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